Analysis of Algorithms

Disjoint Sets

If more than one question appears correct, choose the more specific answer, unless otherwise instructed.

**Concept: disjoint sets as linked lists**

The following questions assume a linked-list implementation of a disjoint set. Assume each value has a pointer to its representative. Assume worst case behavior.

1. The *make-set* operation takes time:
   - (A) constant
   - (B) linear
   - (C) logarithmic
   - (D) log linear

2. The *find-set* operation takes time:
   - (A) none of the other answers are correct
   - (B) log linear
   - (C) linear
   - (D) logarithmic
   - (E) quadratic
   - (F) constant

3. Assuming no preference on which representative becomes the representative of the resulting set, the *union* operation takes time:
   - (A) quadratic
   - (B) none of the other answers are correct
   - (C) logarithmic
   - (D) log linear
   - (E) linear
   - (F) constant

4. Assuming the representative of the smaller set becomes the representative of the resulting set, the *union* operation takes time:
   - (A) logarithmic
   - (B) linear
   - (C) quadratic
   - (D) none of the other answers are correct
   - (E) log linear
   - (F) constant

5. Assuming the representative of the larger set becomes the representative of the resulting set, the *union* operation takes time:
   - (A) log linear
   - (B) logarithmic
   - (C) quadratic
   - (D) linear
   - (E) constant
   - (F) none of the other answers are correct

6. Assuming no preference on which representative becomes the representative of the resulting set, the total work of a series of *m* disjoint set operations is:
   - (A) linear
   - (B) logarithmic
   - (C) log linear
   - (D) constant
   - (E) none of the other answers are correct
   - (F) quadratic

7. Assuming the representative of the smaller set becomes the representative of the resulting set, the total work of a series of *m* disjoint set operations is:
   - (A) constant
   - (B) quadratic
   - (C) log linear
   - (D) linear
   - (E) logarithmic
   - (F) none of the other answers are correct
8. Assuming the representative of the larger set becomes the representative of the resulting set, the total work of a series of \( m \) disjoint set operations is:

(A) linear  
(B) none of the other answers are correct  
(C) constant  
(D) logarithmic  
(E) quadratic  
(F) log linear

9. Assuming the representative of the larger set becomes the representative of the resulting set, how many times can a value's representative be updated in a series of union operations?

(A) linear  
(B) constant  
(C) log linear  
(D) none of the other answers are correct  
(E) logarithmic  
(F) quadratic

10. Assuming the representative of the smaller set becomes the representative of the resulting set, how many times can a value's representative be updated in a series of union operations?

(A) quadratic  
(B) linear  
(C) logarithmic  
(D) log linear  
(E) constant  
(F) none of the other answers are correct

11. Assuming no preference on which representative becomes the representative of the resulting set, how many times can a value's representative be updated in a series of union operations?

(A) none of the other answers are correct  
(B) quadratic  
(C) constant  
(D) log linear  
(E) logarithmic  
(F) linear

**Concept: disjoint sets as trees**

The following questions assume a tree implementation of a disjoint set. Assume each value has a pointer to its parent with the root of the tree serving as the representative of the set. Assume worst case behavior.

12. The **make-set** operation takes time:

(A) linear  
(B) constant  
(C) log linear  
(D) logarithmic

13. The **find-set** operation (no path compression and no union by rank) takes time:

(A) log linear  
(B) quadratic  
(C) linear  
(D) logarithmic  
(E) none of the other answers are correct  
(F) constant

14. The **find-set** operation (with path compression but no union by rank) takes time:

(A) logarithmic  
(B) none of the other answers are correct  
(C) constant  
(D) log linear  
(E) quadratic  
(F) linear

15. The **find-set** operation (with no path compression but with union by rank) takes time:

(A) constant  
(B) none of the other answers are correct  
(C) quadratic  
(D) linear  
(E) log linear  
(F) logarithmic
16. Assuming no preference on which representative becomes the representative of the resulting set, the union operation takes time:

(A) logarithmic  
(B) log linear  
(C) quadratic  
(D) linear  
(E) none of the other answers are correct  
(F) constant

17. Assuming the representative whose tree has the smaller rank becomes the representative of the resulting set, the union operation takes time:

(A) linear  
(B) log linear  
(C) constant  
(D) quadratic  
(E) logarithmic  
(F) none of the other answers are correct

18. Assuming the representative whose tree has the larger rank becomes the representative of the resulting set, the union operation takes time:

(A) quadratic  
(B) linear  
(C) none of the other answers are correct  
(D) constant  
(E) logarithmic  
(F) log linear

19. Assuming no path compression and no union by rank, the total work of a series of $m$ disjoint set operations is:

(A) none of the other answers are correct  
(B) constant  
(C) logarithmic  
(D) linear  
(E) log linear

20. Assuming path compression but no union by rank, the total work of a series of $m$ disjoint set operations is:

(A) none of the other answers are correct  
(B) quadratic  
(C) constant  
(D) linear  
(E) logarithmic  
(F) log linear

21. Assuming no path compression but union by rank, the total work of a series of $m$ disjoint set operations is:

(A) none of the other answers are correct  
(B) logarithmic  
(C) log linear  
(D) linear  
(E) constant  
(F) quadratic

22. Assuming path compression and union by rank, the total work of a series of $m$ disjoint set operations is:

(A) linear  
(B) quadratic  
(C) constant  
(D) none of the other answers are correct  
(E) log linear  
(F) logarithmic

23. Path compression is used to speed up the average cost of which operation(s)?

(A) union and find-set  
(B) find-set and make-set  
(C) make-set, find-set, and union  
(D) union and make-set  
(E) none of the other answers are correct  
(F) find-set  
(G) union  
(H) make-set
24. Union by rank is used to speed up the average cost of which operation(s)?

   (A) union and make-set
   (B) make-set, find-set, and union
   (C) none of the other answers are correct
   (D) make-set
   (E) find-set
   (F) union and find-set
   (G) union
   (H) find-set and make-set

25. T or F: A single find-set operation with path compression takes asymptotically longer than a single find-set operation without path compression, in the worst case.

26. Suppose there are initially \( n \) disjoint sets. If \( m \) union operations are performed, what is the fewest number of disjoint sets that remain? Assume \( n \) is a power of two and \( n > m \).

   (A) \( m \)
   (B) \( n - 2m \)
   (C) \( n - 2^m \)
   (D) \( n - m \)

For the following set of questions, consider the following set of operations:

```plaintext
for each i in 0..9 do make-set(i)
union(1,2);
union(3,4);
union(5,6);
union(7,8);
union(9,0);
union(4,1);
union(4,6);
union(7,9);
find-set(3);
find-set(1);
```

assuming union by rank and path compression. When unioning two sets having the same rank, assume the root with the larger value becomes the root of the resulting set.

27. How many disjoint sets remain?

   (A) none of the other answers are correct
   (B) 5
   (C) 2
   (D) 1
   (E) 4
   (F) 3

28. How many values have a root as parent?

   (A) 1
   (B) none of the other answers are correct
   (C) 4
   (D) 5
   (E) 3
   (F) 2