References to binary search trees, red-black trees, and AVL trees refer to typical implementations, unless otherwise specified.

The terms *niece* and *nephew* come from web page: *The best red-black tree pseudocode ever.*

If more than one question appears correct, choose the more specific answer, unless otherwise instructed.

**Concept: rotations in a BST**

1. Which of the following is true for rotations in a BST:
   - (A) the number of leaf nodes always increases
   - (B) the number of leaf nodes always decreases
   - (C) the tree always becomes more balanced
   - (D) BST-ordering is always preserved

2. Consider a right-rotation of a node $n$ upwards in a BST. The former right child of $n$, if it exists:
   - (A) becomes the right child of the former parent of $n$
   - (B) remains the right child of $n$
   - (C) becomes the sibling of $n$
   - (D) becomes the left child of $n$
   - (E) becomes the left child of the former parent of $n$
   - (F) becomes the niece or nephew of $n$

3. Consider a right-rotation of a node $n$ upwards in a BST. The former left child of $n$, if it exists:
   - (A) becomes the right child of $n$
   - (B) becomes the niece or nephew of $n$
   - (C) becomes the left child of the former parent of $n$
   - (D) becomes the sibling of $n$
   - (E) becomes the right child of the former parent of $n$
   - (F) remains the left child of $n$

4. Consider a right-rotation of a node $n$ upwards in a BST. The former parent of $n$, assuming it exists:
   - (A) becomes the right child of $n$
   - (B) becomes the left child of $n$
   - (C) becomes the sibling of $n$
   - (D) becomes the niece or nephew of $n$
   - (E) remains the parent of $n$

5. Consider a right-rotation of a node $n$ upwards in a BST. The former sibling of $n$, assuming it exists:
   - (A) becomes the right child of $n$
   - (B) becomes the niece or nephew of $n$
   - (C) becomes a grandchild of $n$
   - (D) remains the sibling of $n$
   - (E) becomes the left child of $n$

**Concept: red-black trees**

6. The number of rotations that occur after an insertion into a red-black tree is:
   - (A) $\Theta(\log n)$
   - (B) $\Theta(n)$
   - (C) $\Theta(n \log n)$
   - (D) $\Theta(1)$

7. The maximum number of rotations that occur after an insertion into a red-black tree is:
   - (A) 1
   - (B) $\sim \log n$
   - (C) 3
   - (D) 2

8. The maximum number of rotations that occur after a deletion from a red-black tree is:
   - (A) $\Theta(n)$
   - (B) $\Theta(1)$
   - (C) $\Theta(n \log n)$
   - (D) $\Theta(\log n)$
9. The maximum number of rotations that occur after a deletion from a red-black tree is:
   (A) 3
   (B) $\sim \log n$
   (C) 2
   (D) 1

10. Consider a node $n$ in a red-black tree and all paths from $n$ to a leaf. Which of the following is a constraint on these trees?
   (A) the number of nodes (red or black) on each path is the same
   (B) each path must start with a black node
   (C) the number of red nodes on each path is the same
   (D) the number of black nodes on each path is the same

11. Consider a node $n$ in a red-black tree and all paths from $n$ to a leaf. Which of the following is a constraint on these trees?
    (A) no red node can have a black parent
    (B) no red node can have a red parent
    (C) no black node can have a black parent
    (D) no black node can have a red parent

12. Consider a black interior node $n$ in a red-black tree and any path from $n$ to a leaf. Which of the following is a constraint on these trees, where $R$ is the number of red nodes and $B$ is the number of black nodes?
    (A) $R \leq B$
    (B) $B \leq R + 1$
    (C) $B \leq R$
    (D) $R \leq B + 1$
    (E) $R = B$
    (F) $B < R$

13. Consider a red node $n$ in a red-black tree and the length of the shortest possible path from $n$ to a leaf, $S$, and the length of the longest possible path from $n$ to a leaf, $L$. Which of the following is a constraint on these trees?
    (A) $L = 2S + 2$
    (B) $L = 2S - 1$
    (C) $L = 2S + 1$
    (D) $L = 2S - 2$
    (E) $L = 2S$

14. Inserting a value in a red-black tree and a regular BST, respectively, takes time:
    (A) $\Theta(\log n)$ and $\Theta(\log n)$
    (B) $\Theta(n)$ and $\Theta(n)$
    (C) $\Theta(1)$ and $\Theta(\log n)$
    (D) $\Theta(\log n)$ and $\Theta(n)$

15. Suppose one wished to allow more red nodes in a red-black tree, but still wished this new tree to have the same asymptotic behavior as before. One could allow more red nodes on any path to a leaf as long as:
    (A) no red node could have a red sibling.
    (B) the number of red nodes between any two black nodes is bounded by a constant.
    (C) the number of black nodes between any two red nodes is bounded by a constant.
    (D) no black node could have a red parent.

16. T or F: A black node in a red-black tree can have one child.
17. T or F: A red node in a red-black tree can have one child.
18. T or F: A red node in a red-black tree can have a red parent.
19. T or F: A black node in a red-black tree can have a black parent.
20. Choose an order of insertion for seven consecutive integers such that a red-black tree performs no rotations for any of the insertions.
    (A) 3 2 6 1 0 5 4
    (B) 3 2 6 0 1 4 5
    (C) 3 2 5 0 1 4 6
    (D) 3 2 5 1 0 4 6
    (E) 3 1 5 2 0 4 6
21. Choose an order of insertion for seven consecutive integers such that a red-black tree performs one rotation for one of the insertions and no rotations for the other insertions.

(A) 3 1 5 2 0 4 6  
(B) 3 2 5 0 1 4 6  
(C) 3 2 6 1 0 5 4

(D) 3 2 5 1 0 4 6  
(E) 3 2 6 0 1 4 5

22. Choose an order of insertion for seven consecutive integers such that a red-black tree performs two rotations for one of the insertions and no rotations for the other insertions.

(A) 3 2 6 1 0 5 4  
(B) 3 2 5 1 0 4 6  
(C) 3 1 5 2 0 4 6

(D) 3 2 6 0 1 4 5  
(E) 3 2 5 0 1 4 6

23. Choose an order of insertion for seven consecutive integers such that a red-black tree performs one rotation for two of the insertions and no rotations for the other insertions.

(A) 3 2 6 0 1 4 5  
(B) 3 2 6 1 0 5 4  
(C) 3 2 5 1 0 4 6

(D) 3 1 5 2 0 4 6  
(E) 3 2 5 0 1 4 6

24. Choose an order of insertion for seven consecutive integers such that a red-black tree performs no rotations for any of the insertions and yields the most unbalanced tree.

25. T or F: Inserting the following numbers, in the order given, into an empty BST:

0 4 3 8 1 2 6 5 9 7

yields a tree whose shape is consistent with a red-black tree.

26. Consider inserting the following numbers, in the order given, into an empty BST and then coloring the root black and the other nodes such that no red node has a red parent: each node:

0 4 3 8 1 2 6 5 9 7

What is the minimum / maximum number of red nodes possible?

(A) 0 / 3  
(B) 0 / 4  
(C) 0 / 6  
(D) 3 / 4

(E) the correct answer is not listed  
(F) 0 / 5  
(G) 3 / 3  
(H) 3 / 5

27. Consider an node with a single child in a red-black tree. If that node has a sibling and you wish to maximize the number of descendants the sibling has, what color is the sibling and how many descendants does it have? Do not include the null children. Choose the best answer.

(A) red / 2  
(B) red / 6  
(C) black / 6  
(D) either black or red / 4

(E) black / 2  
(F) either black or red / 2  
(G) the correct answer is not listed  
(H) either black or red / 5

28. Consider inserting the following numbers, in the order given, into an empty red-black tree:

5 3 8 1 7 6 4

How many rotations and how many node recolorings are performed? Don’t forget what happens on the initial insertion.

(A) 0 / 7  
(B) the correct answer is not listed  
(C) 1 / 6  
(D) 0 / 6

(E) 1 / 7  
(F) 0 / 5  
(G) 1 / 5
29. Consider inserting the following numbers, in the order given, into an empty BST:

```
0 4 3 8 1 2 6 5 9 7
```

What is the minimum number of rotations that would yield a tree with a shape consistent with a red-black tree?

(A) 3  
(B) 4  
(C) 0  
(D) 2  
(E) 1  
(F) 5  
(G) the correct answer is not listed

30. Consider inserting the following numbers, in the order given, into an empty red-black tree:

```
0 4 3 8 1 2 6 5 9 7
```

After which insertion value does the red-black tree make its first rotation?

(A) the correct answer is not listed  
(B) 4  
(C) 8  
(D) 0  
(E) 1  
(F) 3  
(G) 2  
(H) 6

31. Consider inserting the following numbers, in the order given, into an empty red-black tree:

```
0 4 3 8 1 2 6 5 9 7
```

Which values, when inserted, cause a rotation?

(A) 6 7  
(B) 2 6 7  
(C) 3 2 6  
(D) the correct answer is not listed  
(E) none  
(F) 3 2 7

32. Consider inserting the following numbers, in the order given, into an empty red-black tree:

```
0 4 3 8 1 2 6 5 9 7
```

Which values, when inserted, cause a double rotation?

(A) 2 6  
(B) the correct answer is not listed  
(C) 3 6  
(D) 7  
(E) 2 7  
(F) 6  
(G) 3 7

**Concept: AVL trees**

33. Consider a node \( n \) in an AVL tree and the height of the left subtree of \( n \), \( LH \) and the height of the right subtree of \( n \), \( RH \). Assuming \( LH > RH \), which of the following is a constraint on these trees?

(A) \( LH - RH < 3 \)  
(B) \( LH - RH = 2 \)  
(C) \( LH - RH < 2 \)  
(D) \( LH - RH = 0 \)  
(E) \( LH - RH = 1 \)

34. The number of rotations that occur after an insertion into an AVL tree is, in the worst case:

(A) \( \Theta(1) \)  
(B) \( \Theta(n \log n) \)  
(C) \( \Theta(\log n) \)  
(D) \( \Theta(\log \log n) \)  
(E) \( \Theta(n) \)

35. The number of rotations that occur after an insertion into an AVL tree is, in the worst case:

(A) \( \Theta(n) \)  
(B) 2  
(C) 1  
(D) \( \Theta(\log n) \)
36. The number of rotations that occur after an deletion in an AVL tree is, in the worst case:
   (A) \( \Theta(n \log n) \)  (B) \( \Theta(1) \)  (C) \( \Theta(n) \)  (D) \( \Theta(\log n) \)  (E) \( \Theta(\log \log n) \)

37. The number of rotations that occur after an deletion into an AVL tree is, in the worst case:
   (A) \( \Theta(n) \)  (B) 2  (C) \( \Theta(\log n) \)  (D) 1

38. The minimum number of rotations that occur after an deletion into an AVL tree is:
   (A) \( \Theta(\log n) \)  (B) \( \Theta(n) \)  (C) 2  (D) 0

39. **T** or **F**: Inserting the following numbers, in the order given, into an empty BST:
   0 4 3 8 1 2 6 5 9 7
   yields a tree whose shape is consistent with an AVL tree.

40. Consider inserting the following numbers, in the order given, into an empty BST and then computing the balance factors of each node:
   0 4 3 8 1 2 6 5 9 7
   What nodes are out of balance, with respect to AVL balance factors?
   (A) 0 1 2 3  (E) the correct answer is not listed
   (B) 2 5 6 7 9  (F) 0 1 3 8
   (C) 0 3  (G) 0 3 8
   (D) 0 4 3

41. Consider an node with a single child in an AVL tree. If that node has a sibling, what is the least / most number of descendants the sibling can have?
   (A) 0 / 7  (F) 1 / 6
   (B) 1 / 4  (G) 0 / 4
   (C) 1 / 5  (H) 1 / 7
   (D) 0 / 5  (I) 0 / 6
   (E) the correct answer is not listed

42. Consider inserting the following numbers, in the order given, into an empty BST and then computing the balance factors of each node:
   0 4 3 8 1 2 6 5 9 7
   Performing a level order traversal of the tree in which balance factors are displayed, which of the following sequences of balance factors is consistent with the above tree? Assume the height of a null child is zero. Do not include the null children in the output.
   (A) -9 -2 2 2 -1 0 0 0 0 0  (D) the correct answer is not listed
   (B) -4 0 2 1 -1 0 0 0 0 0  (E) 1 0 1 1 1 0 0 0 0
   (C) -1 0 1 1 -1 0 0 0 0 0
43. Consider inserting the following numbers, in the order given, into an empty BST and then computing the heights of each node:

   0 4 3 8 1 2 6 5 9 7

Performing a level order traversal of the tree in which node heights are displayed, which of the following sequences of heights is consistent with the above tree? Assume the height of a null child is zero. Do not include the null children in the output.

(A) the correct answer is not listed
(B) 4 4 3 3 2 2 1 1 0 0
(C) 5 5 4 4 3 3 2 2 1 1

44. Consider inserting the following numbers, in the order given, into an empty BST:

   0 4 3 8 1 2 6 5 9 7

What is the minimum number of rotations that would yield a tree with balance factors consistent with an AVL tree?

(A) 5
(B) 1
(C) 3
(D) 4
(E) the correct answer is not listed

45. Consider inserting the following numbers, in the order given, into an empty AVL tree:

   0 4 3 8 1 2 6 5 9 7

After which insertion value causes the AVL tree’s first rotation?

(A) the correct answer is not listed
(B) 1
(C) 5
(D) 3
(E) 9

46. Consider inserting the following numbers, in the order given, into an empty AVL tree:

   0 4 3 8 1 2 6 5 9 7

Which values, when inserted, cause rotations?

(A) 3 7 2 6
(B) 3 7 2
(C) 7 2 6
(D) the correct answer is not listed
(E) 7 6

47. Consider inserting the following numbers, in the order given, into an empty AVL tree:

   0 4 3 8 1 2 6 5 9 7

Which values, when inserted, cause a double rotation?

(A) 7
(B) 6
(C) 2
(D) 3
(E) 2 6
(F) 3 7
(G) the correct answer is not listed

48. Choose an order of insertion for seven consecutive integers such that an AVL tree performs no rotations for any of the insertions.

49. Choose an order of insertion for seven consecutive integers such that an AVL tree performs one rotation for one of the insertions and no rotations for the other insertions.

50. Choose an order of insertion for seven consecutive integers such that an AVL tree performs two rotations for one of the insertions and no rotations for the other insertions.
51. Choose an order of insertion for seven consecutive integers such that an AVL tree performs one rotation each for two of the insertions and no rotations for the other insertions.

52. Choose an order of insertion for seven consecutive integers such that an AVL tree performs no rotations for any of the insertions and yields the most unbalanced tree.