Rough ideas

Consider this Venn diagram:

We can assign the following meanings:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>always worse than lower bound</td>
<td>NSL, WCB</td>
</tr>
<tr>
<td>( o )</td>
<td>always better than upper bound</td>
<td></td>
</tr>
<tr>
<td>( \Omega )</td>
<td>never better than lower bound</td>
<td>NSL, WCB</td>
</tr>
<tr>
<td>( O )</td>
<td>never worse than upper bound</td>
<td>NSL, WCB, ICF</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>never better/worse than upper and lower (tight) bound</td>
<td>NSL, WCB, ICF</td>
</tr>
</tbody>
</table>

under the following conditions:

- the problem size (usually denoted \( n \)) is sufficiently large (NSL)
- we are comparing worst case behavior (WCB)

For the \( \Theta \), \( \Omega \), and \( O \) regions, we apply one more condition:

- we ignore constant factors and other lower order terms (ICF)

If we are comparing the running times of two algorithms, \( f \) and \( g \), we place one of them, say \( g \), in the \( \Theta \) region. If algorithm \( f \) is in the \( \omega \) region, then:

\[ f = \omega(g) \]

The English interpretation is that \( f \) is always slower than \( g \) (NSL, WCB). If \( f \) is in the \( \Theta \) or \( o \) regions, then:

\[ f = O(g) \]

The English interpretation of this statement is that \( f \) is never slower than \( g \) (NSL, WCB, ICF).

When using order notation, one generally assumes that NSL, WCB, and ICF always apply, unless otherwise stated.

**Formal Statements**

A formal definition of \( O \) is:
if $\lim_{n \to \infty} \frac{f}{g} = 0$

The definitions of the other symbols are similar.